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On-line Identification of the DC motor Parameters by using Least Mean Square Recursive Method

The on-line least square parameters (electrical and mechanical) identification method of the separately DC motor is presented in this paper. The parameters of the DC machine are time dependent; therefore, for the DC drive control the on-line parameters identification procedure is mandatory. By knowing accurately the DC motor parameters, the parameters of the DC speed and current cascade controllers can be obtained.

Keywords: DC motor, least mean square, identification, recursive

1. Introduction

On-line identification algorithms derives from off-line one, such that the estimate vector parameters $\hat{\theta}$ (N +1) based on the N +1 data can be recursively calculated taking into account the several previous measurements data. Additional task has to be included in the on-line recursive identification method. can be transformed into a recursive form so that the information obtained from the last sample to be more influential than the algorithm came from previous samples. In this way, a "forgetting factor" appears which is very useful in identifying of the time varying parameters. In order to implement the proposed identification algorithm the Matlab/Simulink programming environment has been used. Following the numerical simulation of electrical and mechanical parameters of the DC machine obtained through the identification process will be compared with the actual parameters of the separately excited DC motor. Based on the estimated mechanical and electrical parameters of the DC motor, the digital controllers are adjusted in real-time. The above mentioned argument fully justifies the study of methods for estimating on-line the parameters of electrical machines. Estimation technique chosen for this work is based on the method of least squares recursive version. Least mean square algorithm is the most used in adaptive filtering [1]-[5], due to the low computational necessity and high stability [2]-[5].

2. Mathematical model of the separately excited DC motor

To address this algorithm for estimating equations the mathematical model of the DC motor operating at constant flux are described:

$$
\begin{aligned}
u &= Ri(t) + L\frac{di(t)}{dt} + K\,\mathrm{t}\,(t) \\
m &= K\,i(t) = J\,\frac{d\,\mathrm{t}\,(t)}{dt} + Kf\;\mathrm{t}\,(t) + T_L
\end{aligned}, \tag{1}
$$

in which:

- $u(t)$ - rotor voltage;
- $i(t)$ - rotor current;
- $\mathrm{t}\,(t)$ - angular velocity of the rotor;
- R - rotor resistance;
- L - rotor inductance;
- K - DC motor constant;
- Kf - viscous friction constant;
- J - moment of inertia of the rotor;
- T_L - the load torque;

By dividing the first equation of (1) system by L and the second by J the standard steady state form of a linear DC motor dynamic system can be obtained:

$$
\begin{aligned}
\frac{di(t)}{dt} &= -\frac{R}{L}i(t) - \frac{K}{L}\,\mathrm{t}\,(t) + \frac{1}{L}u(t) \\
\frac{d\,\mathrm{t}\,(t)}{dt} &= \frac{K}{J}i(t) - \frac{f}{J}\,\mathrm{t}\,(t) - \frac{1}{J}m_r
\end{aligned}, \tag{2}
$$

or in standard state space form

$$
\frac{\mathrm{d}}{\mathrm{dt}}\begin{bmatrix} \mathrm{i(t)} \\ \Omega(\mathrm{t}) \end{bmatrix} = \begin{bmatrix} -\frac{\mathrm{R}}{\mathrm{L}} & -\frac{\mathrm{K}}{\mathrm{L}} \\ \frac{\mathrm{K}}{\mathrm{J}} & -\frac{\mathrm{f}}{\mathrm{J}} \end{bmatrix}\begin{bmatrix} \mathrm{i(t)} \\ \Omega(\mathrm{t}) \end{bmatrix} + \begin{bmatrix} \frac{1}{\mathrm{L}} & 0 \\ 0 & -\frac{1}{\mathrm{J}} \end{bmatrix}\begin{bmatrix} \mathrm{u(t)} \\ \mathrm{m_r} \end{bmatrix}, \tag{3}
$$

If the system (10.3) is discretized with T sampling period, the specific rotor current $i_k = i(kT)$ and angular velocity $\mathrm{t}_k = \mathrm{t}\,(kT)$ samples of k-th order can be noted, $i_k = i(kT)$ and $\mathrm{t}_k = \mathrm{t}\,(kT)$ respectively. The new discretized system equations can be written in the adequate form of the required least squares algorithm:

$$[u-E]=\begin{bmatrix} i_k & \frac{di_k}{dt} \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$[T]=\begin{bmatrix} \frac{d\omega_k}{dt} & 1 & \omega_k \end{bmatrix}\begin{bmatrix} \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}, \quad (4)$$

where the following additional notations have been made:

$$\theta_1 = R_a \qquad \theta_2 = L_a$$
$$\theta_3 = J \qquad \theta_4 = T_L \qquad \theta_5 = Kf \; . \quad (5)$$

The marked expressions (5) form the set of electrical and mechanical parameters to be determined (R, L_a, J, T_L, Kf). The first equation of the (4) system includes the electrical parameters and the second one the mechanical parameters, both equations being linear in θ_i terms.

3. Recursive least mean squares identification algorithm applied to DC motor

The discrete mathematical model of the DC motor can be described in terms of input u(t) and output y(t) with the adequate order of the coefficients A, B as:

$$Ay(t) = Bu(t-1) \quad (6)$$

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-n_a}$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{nb} z^{-n_b} \quad (7)$$

or in the form:

$$y(t) = x^T \theta \quad (8)$$

θ_i - vector of unknown parameters.

The output of the mathematical model can be considerate as:

$$y(t) = x^T \hat{\theta} + \hat{e}(t) \quad (9)$$

$\hat{\theta}$ -the identified values of the parameters of the system;

$\hat{e}(t)$ -modelling (prediction) error

From (8) and (9), the modelling error can be extracted:

$$\hat{e}(t) = x^T\left(\theta - \hat{\theta}\right). \tag{10}$$

By choosing an adequate performance function [1], based on the square prediction error:

$$J = \sum_{1}^{N} \hat{e}^2 = \hat{e}^T \hat{e}\,, \tag{11}$$

the unknown parameters of the system are determined as a solution to:

$$\frac{\partial J}{\partial \hat{\theta}} = 0\,. \tag{12}$$

Taking into consideration eqs (11) and (12), the adequate solution can be found

$$\hat{\theta} = \left[X^T X\right]^{-1}\left[X^T y\right]\,. \tag{13}$$

By measuring the input and output signals of the system the recursive least squares method can be drawing as in Fig. 1.

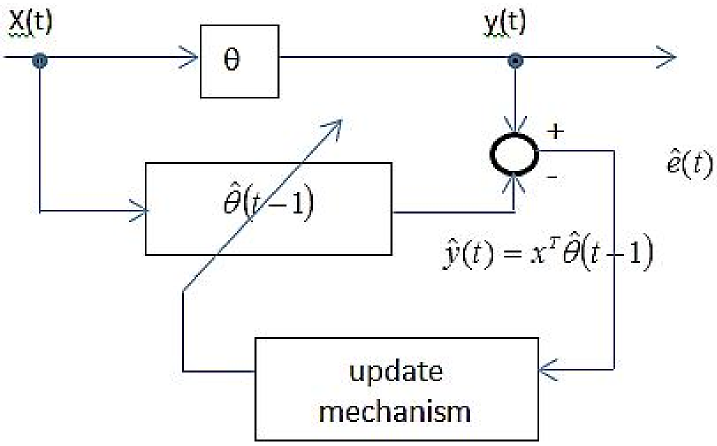

Figure 1. Recursive least mean squares method [1].

By using the least squares method the vector of the estimated parameters can be provided:

$$\hat{\theta} = \left[\sum_{i=1}^{N} \chi_i \chi_i^T\right]^{-1}\left[\sum_{i=1}^{N} \chi_i y_i\right], \tag{14}$$

where for the electrical equation, $[u - E]$ is considerated as y_i, and the term $[T]$ (4) denotes the mechanical components. In the same manner the following terms are defined:

$$c_i^T = \left[i_k \quad \frac{di_k}{dt} \right], \qquad c_i^T = \left[\frac{d\,t_k}{dt} \quad 1 \quad t_k \right] \tag{15}$$

and

$$\hat{\theta}_e = [\theta_1 \quad \theta_2], \tag{16}$$

$$\hat{\theta}_m = [\theta_3 \quad \theta_4 \quad \theta_5]. \tag{17}$$

In practice very useful is the recursive version of this algorithm, so that at the current sampling time the parameters depend on the values from the previous sampling time. The estimation method can be transformed into a recursive form so that the information coming from the last sampling period to be more influential than that coming from the previous samples. This cause of using to a "forgetting factor" marked by a_k that it is very useful in identifying of time varying parameters.

Suppose now that the DC motor parameters are identified on a a short time (eg starting). the used algorithm to identify the mechanical and electrical parameters is based on the recursive least squares:

$$\begin{aligned} \hat{\theta}(k) &= \hat{\theta}(k-1) + L(k)\varepsilon(k) \\ \varepsilon(k) &= y(k) - \Phi^T(k)\hat{\theta}(k-1) \\ L(k) &= \frac{P(k-1)\Phi(k)}{1 + \Phi^T(k)P(k-1)\Phi(k)} \end{aligned} \tag{18}$$

$$P(k) = P(k-1) - \frac{P(k-1)\Phi(k)\Phi^T(k)P(k-1)}{1 + \Phi^T(k)P(k-1)\Phi(k)}$$

Obviously, recursive algorithm requires knowledge of the initial values of the $\hat{\theta}(0)$ parameters and matrix P (0).

Interpretation of relations (18) is relatively simple. New estimates $\hat{\theta}(k)$ are obtained by applying the old one, $\hat{\theta}(k-1)$, adding a proportional correction, L(k), with the error ε(k). At the same time ε(k) represent the optimal step prediction error (the difference between the measured output at time k (y (k)) and predicted $\Phi^T(k)\hat{\theta}(k-1)$. The correction coefficient L(k) is modified appropriately at each sampling step.

Simulation of the DC motor with the recursively estimation algorithm is presented in Figure 2:

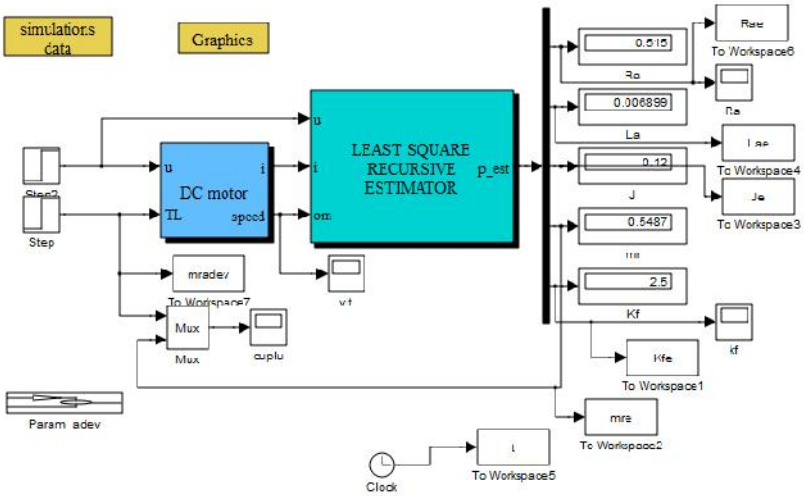

Figure 2. Simulink simulation diagram based on the least squares recursive estimation algorithm

In the Fig. 2 Simulink block of the recursively estimated parameters is shown. It uses a multiplexer to gather information of the rotor current, DC motor speed, their derivatives and the rotor voltage.

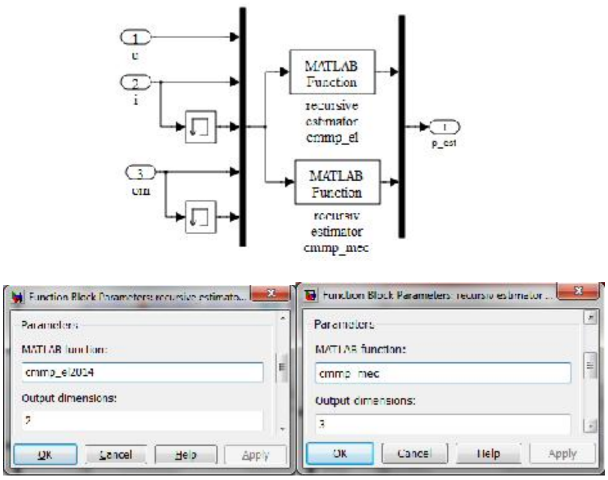

Figure 3. On-line identification of the DC motor parameters

In order to find the electrical and mechanical parameters of the DC motor two Matlab function have been designed. Based on the equations system 18 the recursive estimation algorithm is implemented. In order to find the electrical parameters, in the Fig. 4 the built-in Matlab function that implements the on-line recursive least squares estimation method is shown:

```
function tetael=cmmp_el2014(in)

global tetael_an Pel_an lambda_el Kfi T

y=in(1)-Kfi*in(4);

fi=[in(2);(in(2)-in(3))/T];
denum_el=lambda_el+fi'*Pel_an*fi;
L=(Pel_an*fi)/denum_el;
tetael=tetael_an+L*(y-fi'*tetael_an);
Pel=(Pel_an-
(Pel_an*fi*fi'*Pel_an)/denum_el)/lambda_el;
tetael_an=tetael;
Pel_an=Pel;
```

Figure 4. On-line electrical parameters (R_a, L_a) recursive least squares (RLS) identification method based on the buit-in Matlab function.

```
function tetamec=cmmp_mec(in)

global tetamec_an Pmec_an lambda_mec alpham denumm Pm Kfi
T

y=Kfi*in(2);

fi=[(in(4)-in(5))/T;1;in(4)];
denumm=(lambda_mec+fi'*Pmec_an*fi);
L=(Pmec_an*fi)/denumm;
alpham=(y-fi'*tetamec_an);
tetamec=tetamec_an+L*alpham;
Pm=L*fi'*Pmec_an;
Pmec=(Pmec_an-Pm)/lambda_mec;
tetamec_an=tetamec;
Pmec_an=Pmec;
```

Figure 5. On-line mechanical parameters (J, T_L, kfi) recursive least squares identification method based on the buit-in Matlab function

The mechanical parameters are identified in the same manner by using the designed Matlab function (Fig.5).

An important issue in practice is the time varying parameters during operation (resistance increases with temperature, the inductance depends on the saturation, the moment of inertia can change with the variation of mechanical load, etc). The on-line parameters estimation taking into account only the information received from the recently sample values of the state measurements is necessary. This task is performed by introducing a forgetting factor λ in the equations system (18), which may be constant or may vary depending on certain criteria. The choosen case is with constant value.

4. Simulation results

The obtained electrical and mechanical parameters are as follows:

$\hat{R}_a = 0.515\ \mathrm{t}$, $\hat{L}_a = 0.006899\mathrm{H}$, $\hat{J} = 0.12 kgm^2, T_L = 0.5487\mathrm{s}, \hat{K}_f = 2.5$.

The real parameters data:

R_a=0.515Ω; L_a=6.9*1e-3H; J=0.12kgm^2; T_L=0.5s, viscous force K_f=2.5.

Estimated parameters were recorded during the simulation, Figure 5. Note the good convergence of the method, the time required to achieve stability is very short (about 100 steps to the electrical parameters).

The initial data values:

- P_el matrix: $100 \cdot \mathbf{I}_n$;
- electrical parameter vector: tetael_an=[0 0]'; .
- mechanical parameter vector: tetamec_an=[0 0 0]';
- sampling time: T=0.000001;

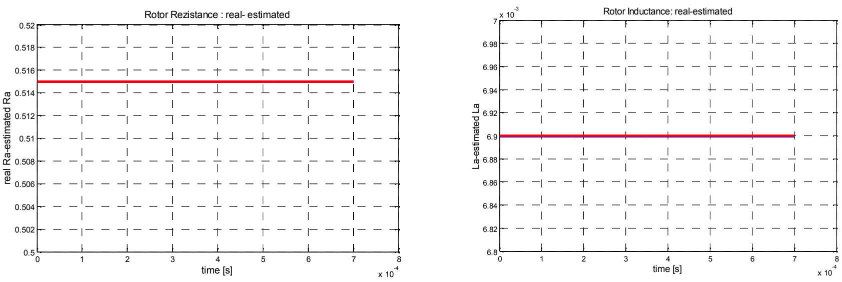

Figure 6. Electrical parameters (Ra, La) identified by using RLS method

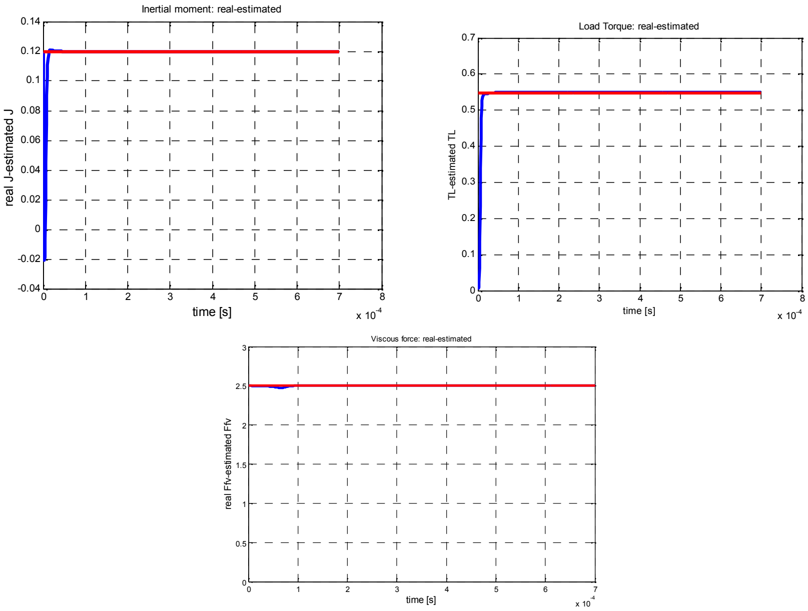

Figure 7. Mechanical parameters (J, TL, kf) identified by using RLS method

4. Conclusions

The electrical parameters are determined accurately very fast. The estimated mechanical parameters are slightly different from the real ones. The interval of the mechanical parameters variations is large at the first 100 sample time due to the fact that the initial parameters are not closely to the real ones. However the speed of convergence depends on the initialization of the recursive algorithm by values $\hat{\theta}(0)$ and P (0). These initial values may be provided, for example, from a previous estimate of the system. In general, if there is no pre-requisite information on the initial values, the natural choice is $\hat{\theta}(0)$ = 0. To express great uncertainty of this initialization, the matrix P (0), which is proportional to the covariance matrix of the estimator, must have elements on the main diagonal high. Frequently is using P (0) = cI with c >> 0. In practice the value of c cannot be chosen too large to not cause numerical instability. A corresponding amount is deducted experimental c = 100 M[y^2(t)].

$$\hat{a}^2 = \frac{1}{N}\sum_{t=1}^{N}\left[y(t)-\varphi^{T}(t)\hat{\theta}(N)\right]^2 = \frac{1}{N}V(N)$$

If $c \to \infty$, the recursive estimator, initialized properly, lead to the same parameter values obtained with the off-line estimator.

If the recursive algorithm converges to the true values of the parameters, the choice of the initial value has no influence. However, the choice of initial values will significantly influence the transient behavior of the algorithm.

Since recursive estimator of the least squares estimator converges to off-line estimated parameters if it is properly initialized, it follows that it is asymptotically normally distributed with mean equal to the true parameters and covariance matrix:

$$\operatorname{cov}\hat{\theta}(k) = \lambda^2 P(k).$$

If the matrix P (N) can be appreciated at every step, dispersion function, λ^2, is generally unknown. It can be replaced with an estimate of its data based on the N available data:

$$\hat{\lambda}^2 = \frac{1}{N}\sum_{t=1}^{N}\left[y(t) - \varphi^T(t)\hat{\theta}(N)\right]^2 = \frac{1}{N}V(N)$$

Thus at every step we are able to appreciate the precision of the estimator. However, this assessment is cumbersome if V (N) is not recursively computed. Criterion function V (N) satisfies the following recurrence relation:

$$V(N+1) = V(N) + \frac{\varepsilon^2(N+1)}{1 + \varphi^T(N+1)P(N)\varphi(N+1)}$$

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References

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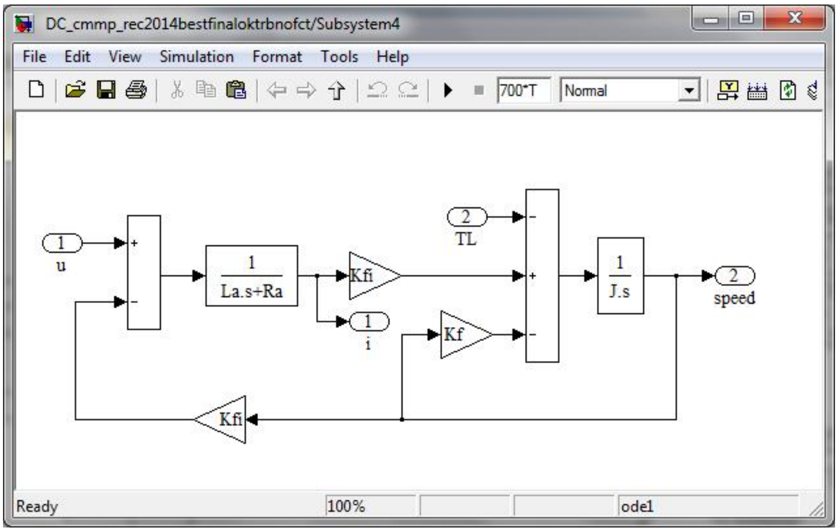
Appendix

```
clear all; clc
% real motor data
Ra=0.515; La=6.9*1e-3; J=0.12;
global Kfi
% constant flux
Kfi=0.765;
%viscous force
Kf=2.5;
Mr=0.5;
% sampling period
global T
T=0.000001;
global Pel_an tetael_an lambda_el
Pel_an=1000*eye(2);
tetael_an=[Ra La]';
lambda_el=0.6;%lambda_el=0.98;
global Pmec_an tetamec_an lambda_mec
Pmec_an=100*eye(3);
tetamec_an=[J Mr Kf]';
lambda_mec=0.79;
```

% Simulink model of the separately excited DC motor at constant flux


DC_cmmp_rec2014bestfinaloktrbnofct/Subsystem4
File Edit View Simulation Format Tools Help
700*T
Normal
1
u
+
-
1
La.s+Ra
Kfi
1
i
2
TL
Kf
1
J.s
2
speed
Kfi
Ready
100%
ode1

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