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### Considerations Concerning Matrix Diagram Transformations Associated with Mathematical Model Study of a Three-phase Transformer

The clock hour figure mathematical model of a three-phase transformer can be expressed, in the most plain form, through a 3X3 square matrix, called code matrix. The lines position reflect the modification in the high voltage windings terminal and the columns position reflect the modification in the low voltage winding terminal. The main changes on the transformer winding terminal are: the circular permutation of connection between windings; terminal supply reversal; reverse direction for the phase winding wrapping; reversal the beginning with the end for a phase winding; the connection conversion from N in Z between phase winding or inverse. The analytical form of these changes actually affect the configuration of the mathematical model expressed through a transformations diagram proposed and analyzed in two ways: bipolar version and unipolar version (fanwise). In the end of the paper are presented about the practical exploitation of the transformations diagram.

Keywords: clock hour figure, transformation diagram, mathematical model

#### 1. Introduction

The clock hour figure mathematical model of a three-phase transformer can be expressed, in the most plain form, through a 3X3 square matrix, called code matrix and whose configuration changes based on the associated clock hour figure. Studies reveal the interdependence between the code matrix configuration (lines and columns position) and the terminal connection configuration or the phase winding connection [7], [11].

In general, the lines position reflects the high voltage winding and connection modification, and the columns reflect the low voltage winding and connection modification, as suggested in figure 1.

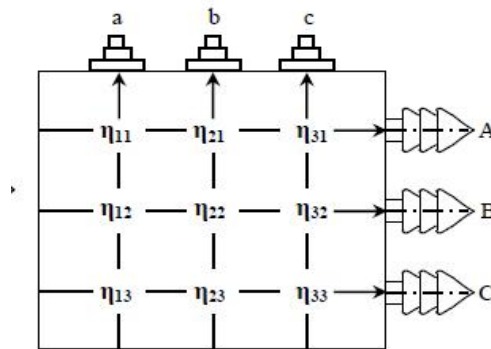


Figure 1 The relationship between the configuration of matrix code and the connection configuration of the transformer terminals [7],[11]

The concern of the authors in these paper lies in the identification of a synthetic and comprehensive, at the same time, method which can be faithfully highlighted all the modification on the connection diagram of a three-phase transformer for the clock hour figure identification.

This object is based on the observation that code matrices reflect faithfully the major modification of the transformer, as follows [3], [6], [7], [11]:

- the circular permutation of connection between windings;
- terminal supply reversal (high voltage winding or low voltage winding);
- 3I reversal (reverse direction for the phase winding wrapping, reverse the beginning with the end, reverse the phase winding connection);
- reverse the winding terminal connection;
- the conversion from N in Z or Z in N for the windings in zigzag or triangle connection.

The idea if a transformation diagram was suggested by the solution obtained for the terminal supply reversal on a three-phase transformer (from the high voltage winding to low voltage winding).

In terms of clock hour figure, by international convention, it is considered that all transformers are step – down voltage (supplied on the high voltage winding). Accordingly, the high voltage winding is considered to be the reference element. In this case, the phase angle expressed by the clock hour figure „i“, is typically the phase shift between the primary and secondary correspondent circuit voltage, for example, between  $\underline{U}_{AB}$  and  $\underline{U}_{ab}$ .

In practice there are cases that require the phase angle „ i ” assessment, considering the low voltage winding as reference. These can be exemplified by the case of a 10MVA transformer, 10/6,6 KV which can operate as a booster transformer on a line system, or as a reducing transformer.

The power supply reversal for a threephase transformer is presented in figure 2.

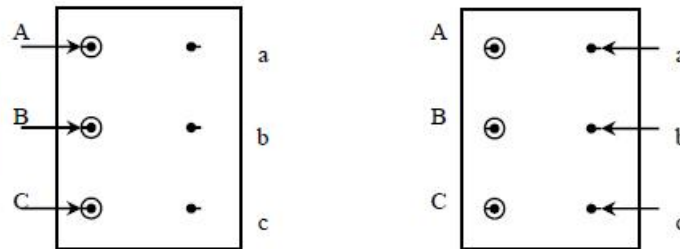


Figure 2 The transformer power supply on the high voltage winding respectively on the low voltage winding [7], [11]

The referred modification is reflected in the code matrix configuration, being achieved the modification of the lines with columns as in figure 3.

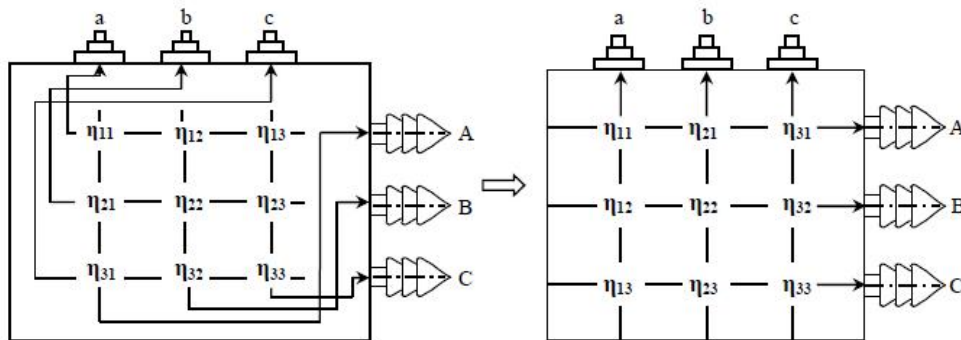


Figure 3 The code matrix configuration for the transformer power supply reversal [7], [11]

The matrix equation that indicate the resulted clock hour figure are:

$$\begin{array}{ll}
 G_1^T = G_{11}; & G_7^T = G_5 \\
 G_2^T = G_{10}; & G_8^T = G_4 \\
 G_3^T = G_9; & G_9^T = G_3 \\
 G_4^T = G_8; & G_{10}^T = G_2 \\
 G_5^T = G_7; & G_{11}^T = G_1 \\
 G_6^T = G_6; & G_{12}^T = G_{12}
 \end{array} \quad (1)$$

These modification can be expressed graphically as graphic construction shown in figure 4.

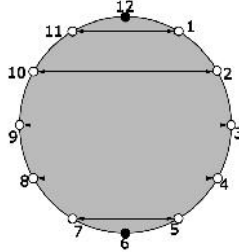


Figure 4 Explanatory for the clock hour figure modification for the power supply reversal [7], [11]

These constructions are the starting point in building the diagram transformation concept, capable of expressing all the connection diagram transformation for a three-phase transformer aiming the modification of the initial clock hour figure.

Authors studies highlight two possibilities for the transformation diagram construction:

- parallel sequence diagram;
- fan sequence diagram.

## 2. Contribution at the parallel sequence diagram

The transformation diagram achieved in the parallel sequence is constituted from six classes illustrated in the table 1.

Table 1.

<p>Class 1</p>	<p>Class 2</p>	<p>Class 3</p>
<p>Class 4</p>	<p>Class 5</p>	<p>Class 6</p>

Each class consists of a circle and in its circumference are placed at equidistant intervals twelve indices and the mentioned circle is associated with a polar axis joining (two clock hour figure), in diametrically opposite position.

The representation is associated with five sequences plotted by some lines connecting each two clock hour figure that can be converted into each other. These lines are parallel to each other and perpendicular to the associated polar axis [12], [13].

The sequence 1 – 11 illustrated in figure 5 from the class 1 are then analyzed.

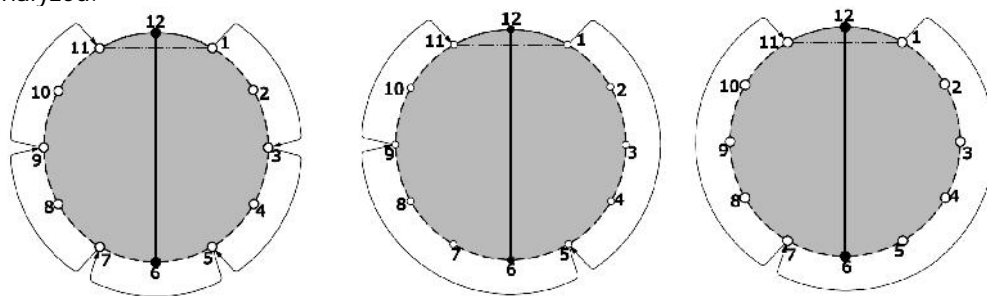


Figure 5. Transformation diagram class 1 – ssequence 1 – 11

According to this construction the 1 – 11 index conversion may be accomplished by different routs as follows:

- a straight line represented by the bisecant that reunite the points 1 – 11 and is distinctly marked by : ( — · · — · · — );
- a short curvilinear lay - out represented by a short arc that reunite on the circle circumference the points 1 – 11 and is distinctly marked by: ( — — — — — );
- a long curvilinear lay - out represented by a long arc that reunite on the circle circumference the points 1 – 11 and is distinctly marked by: ( — — — — — ).

The conversion  $1 \rightarrow 11$  or  $11 \rightarrow 1$ , on the straight lay - out necessarily involves on the initial code matrix, a transposition followed by other changes (circular permutation, reverse on a pairs of lines followed by a reversal to a pair of columns, sign inversion or combinations of inversions and permutations).

The conversion  $1 \rightarrow 11$  or  $11 \rightarrow 1$ , on the curvilinear lay - out involves, necessarily, on the initial code matrix, a circular permutation of rows or columns followed by other changes (inversions, inversions and permutations associations etc). In figure 5 is analysed the conversion  $1 \leftrightarrow 11$  on the three routes previously mentioned.

1. Rectilinear lay – out

$$G_1 = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix}^T \rightarrow \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} = [G_1]^T = G_{11} \quad (2)$$

2. Short curvilinear lay - out

$$G_1 = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix} \quad G_{11} = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\left[ \left[ \left[ \vec{G}_1 \right] \right]_{invab} \right]_{invBC} = G_{11} \quad (3)$$

3. The long curvilinear lay – out in first variant is constituted through the steps: 1 – 3 step; 3 – 5 step; 5 – 7 step; 7 – 9 step; 9 – 11 step

$$2 \cdot \left[ 2 \cdot \left[ 2 \cdot \left[ 2 \cdot \left[ 2 \cdot [G_1] \uparrow \right] \uparrow \right] \uparrow \right] \uparrow \right] \uparrow = G_{11} \quad (4)$$

4. The long curvilinear lay – out in second variant is constituted through the steps: 1- 5 step; 5 – 9 step; 9 – 11 step

$$2 \left[ \left[ \left[ \vec{G}_1 \right] \downarrow \right] \downarrow \right] \downarrow = G_{11} \quad (5)$$

5. The long curvilinear lay – out in third variant is constituted through the steps: 1–7 step; 7-11 step

$$\left[ 2 \cdot [G_1] \right] \downarrow = G_{11} \quad (6)$$

### 3. Contribution to the fanwise sequence diagram

The fanwise sequence transformation diagram is constituted in a first variant from six subclasses associated with the odd clock hour figure (see Table T2) and six subclasses associated with the even clock hour figure. Each subclass is made up of clock hour figure circle, on the circumference of which is placed a radiant pole that leaves on several rectilinear lay – outs and having on the tail end the clock hour figure resulting from the conversion. In other variant Table T4, the fanwise sequence transformation diagram is realised in bipolar solution by overlapping two radiant poles spectrum related in opposite positions.

Table 2.

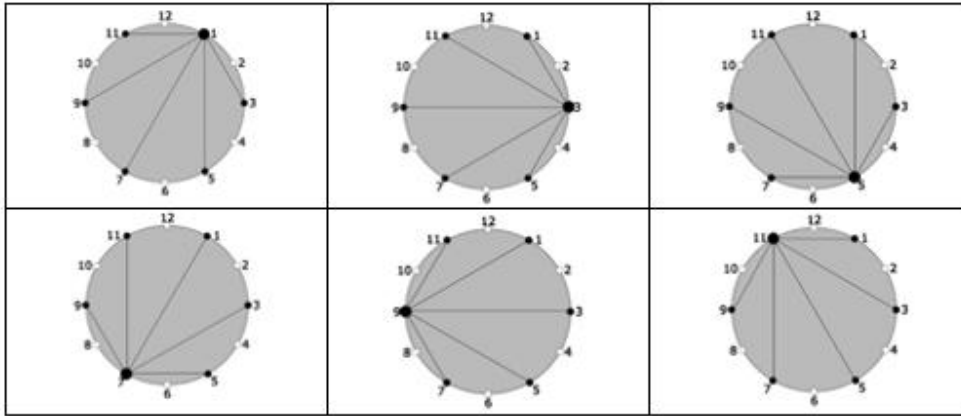


Table 3.

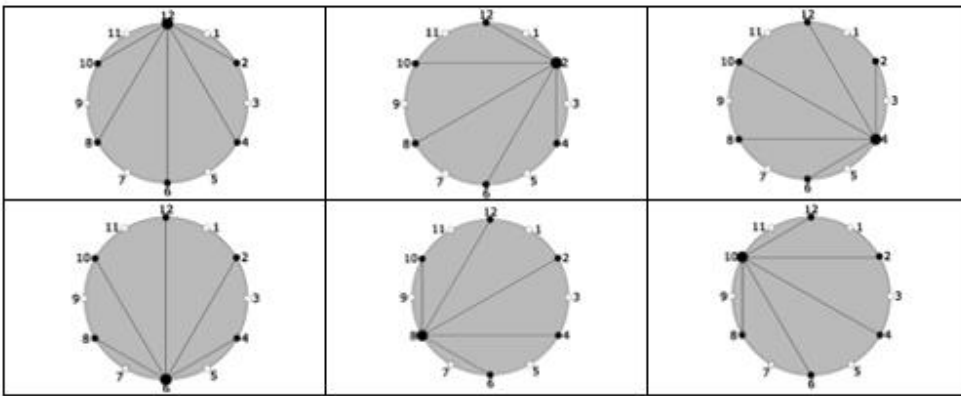
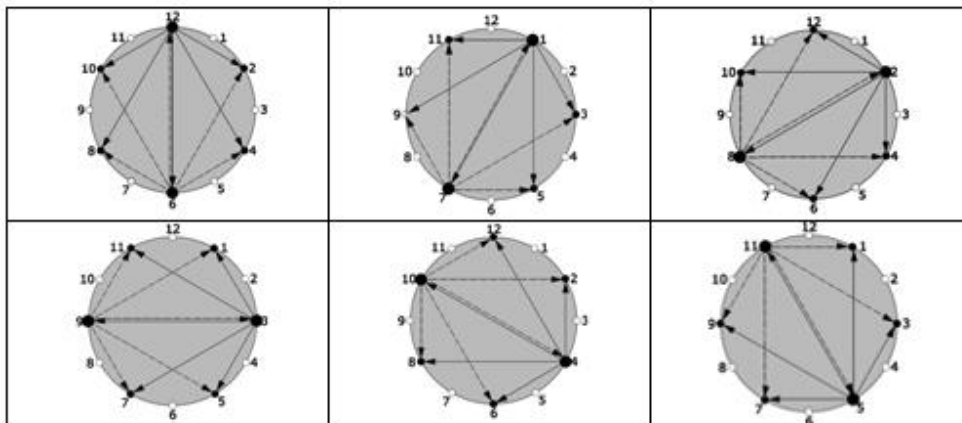


Table 4.



The following describes as an example of one subclass of the sequence  $1 \rightarrow 3$  (figure 6) and as in parallel sequence diagrama the clock hour figure conversion  $1 \rightarrow 3$  la  $3 \rightarrow 1$  can be achieved by three different routes as follows:

- a rectilinal lay - out;
- a short curvilinear lay - out;
- a long curvilinear lay - out.

The representation method is the same as that used for mapping the parallel sequences.

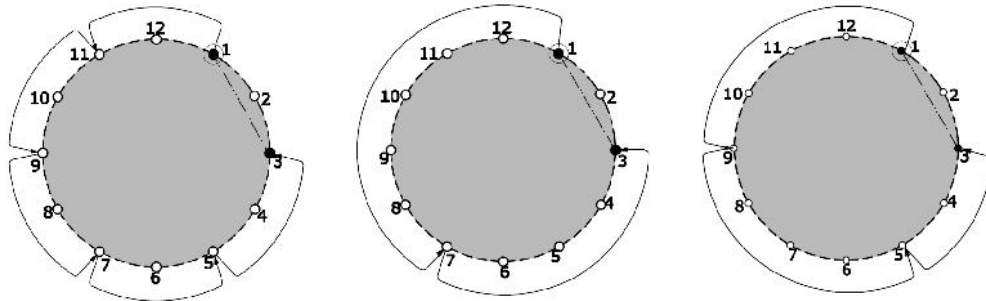


Figure 6. Transformation diagram class1 – sequence 1 – 11

In figure 6 is analysed the conversion  $1 \leftrightarrow 3$  on the three routes previously mentioned.

1. The rectilinal lay - out

$$G_1 = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix} \quad G_3 = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

$$\left[ \left[ \overset{\leftarrow}{G_1} \right]^T \right] = G_3 \quad (7)$$

2. The short curvilinear lay - out

$$\left[ \left[ \left[ \overset{\rightarrow}{G_1} \right] \right]_{inv\mathbf{bc}} \right]_{inv\mathbf{BC}} = G_3 \quad (8)$$

3. The long curvilinear lay – out in first variant is constituted through the steps:: 1 – 11 step; 11 – 9 step; 9 – 7 step; 7 – 5 step; 5 – 3 step



$$2 \cdot \left[ 2 \cdot \left[ 2 \cdot \left[ 2 \cdot \left[ 2 \cdot \left[ 2 \cdot \left[ G_1 \right] \right] \right] \right] \right] \right] = G_3 \quad (9)$$

4. The long curvilinear lay – out in second variant is constituted through the steps: 1- 9 step; 9 – 5 step; 5 – 3 step

$$2 \left[ \left[ \left[ \left[ G_1 \right] \right] \right] \right] = G_3 \quad (10)$$

5. The long curvilinear lay – out in third variant is constituted through the steps: 1–7 step; 7-3 step

$$\left[ 2 \cdot \left[ G_1 \right] \right] = G_{11} \quad (11)$$

#### 4. Conclusion

1. The transformation diagram as conceived and presented in this paper offers the possibility to highlight all the possibilities of modification the afferent three-phase transformer clock hour figure [7], [11].

2. The basic rule underlying the design and use of diagram transformation is the fact that it can be converted into each other only two indices belonging to the same group (even indices group and odd indices group).

The diagram transformation is constituted from several classes, and each class is constituted from the clock hour figure circle where are presented the fundamental poles and the afferent sequences for each classes.

3. The authors have designed the diagram transformation by two solutions: parallel sequence diagram and fanwise sequence diagram.

4. The parallel sequence diagram is characterized by two main poles placed diametrically opposite on the clock hour figure circle, and which are joined by a polar axis regarding that the parallel sequences are perpendicular to each other.

5. The diagram transformation realised in solution with fan sequences are characterized by a radiant pole on the clock hour figure circle and from where are leaving straight lay – outs that connect this pole with the indices resulted from the conversion and placed on the clock hour figure circle.

6. As is described, are obtained six subclasses associated with odd indices and six subclasses associated with even indices. The two types of subclasses can be combined into a common variant comprising only six classes, each one resulted from the overlapping the radiant spectrum corresponding with two radiant poles placed in diametrically opposite positions.

7. The transformation made by rectilinear lay – out necessarily involves a transposition of rows and columns of the code matrix, while the transformation made by the curvilinear lay – out involves a circular permutation of lines or columns of matrix in direct or reverse direction, depending on real conversion conditions.

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