ANALELE UNIVERSITĂȚII
"EFTIMIE MURGU" REȘIȚA
ANUL XXIII, NR. 1, 2016, ISSN 1453 - 7397

Contributions to the Kinematic Analysis of the Double Harmonic Gear Transmission

Draghiţa Ianici, Sava Ianici

The paper presents the kinematic analysis of the double harmonic gear transmission by comparative determining of the transmission ratio for two constructive variants of these transmissions. It demonstrates the viability of double harmonic gear transmission (DHGT) by presenting the operating principle and calculation of transmission ratio by using the analytical method of inversion motion. We performed a comparative analysis of transmission ratio for the two kinematic models of DHGT ($z_2 = z'_2$, respectively $z_2 \neq z'_2$).

Keywords: *double harmonic gear transmission, kinematic analysis, transmission ratio, kinematic model, flexible toothed wheel*.

1. Introduction

Continuous modernization and diversification of constructions and actuators of the industrial robots resulted in improvement of existing mechanical transmissions, and at the occurrence of new gear transmissions, better performing.

In this context, a special attention was given to the study and research of harmonic gear transmission, which is essentially different from conventional gear, because it allows transmission of rotation motion by propagation of elastic deformations (named waves) on the periphery of a flexible element, with some frequency, according to a harmonic law [1], [2], [3], [4], [7], [8].

The double harmonic gear transmission (DHGT) is included in the category of new harmonic gear transmission; this type of transmission presents the following advantages [5, 9]: compact, modular and coaxial constructions, a small axial dimension and low weights, a silent operation, a very high kinematic precision, a high transmission ratio on a single stage (i = 40…150), a high mechanical efficiency at high transmission ratio, a low inertia moment, etc.

Harmonic gear transmissions are used in branches of top technique, namely in construction of: industrial robots, spacecraft, nuclear reactors, computers, radar antennas, airplanes and helicopters, precision dividing heads, naval mechanisms, cranes, servo-mechanisms, gearmotors, actuators in hermetic spaces from chemical and petroleum industry, etc [1, 8, 9, 12, 13].

2. Construction and functioning of DHGT

DHGT (Figure 1) is used in the next functional structure: 1 - the waves generator with cam as a input element, 2 - the flexible toothed wheel with external teeth (z_2), respectively inner teeth (z'_2) at the two ends, as a intermediate element, 3 - the rigid wheel with internal teeth (z_3) as a fixed element, and 4 - the rigid mobile wheel with external teeth (z_4) as a output element [6, 9, 10, 11].

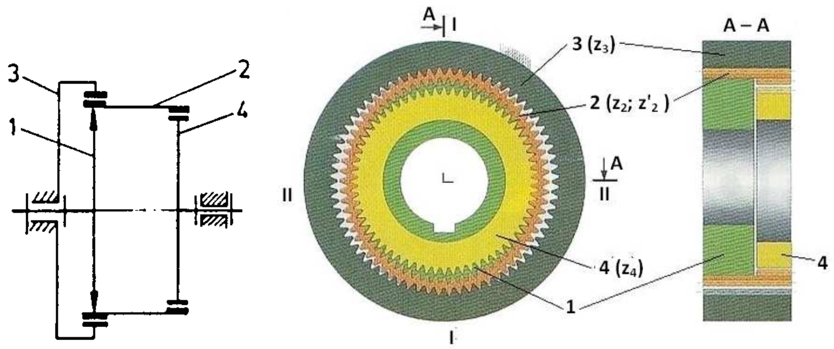

Figure 1. Double harmonic gear transmission

Functioning of the DHGT is similar to the simple harmonic gear transmission. The waves generator (1) is forcely mounted inside the flexible toothed wheel (2) which it deforms elliptically, so that we can distinguish four equidistant drive zones: two with the rigid fixed wheel (3) – representing the first step of harmonic drive (z_3 / z_2 – the left frontal plan of the flexible wheel), and two with the rigid mobile wheel (4) – representing the second step of harmonic drive (z'_2 / z_4 – the right frontal plan of the flexible wheel). Between the two pairs of opposing harmonic drive zones (I-I and II-II) we have an angle of 90^o.

The forced elliptical deformation of the flexible toothed wheel must ensure the input of the external teeth of the flexible wheel (2) in gaps between the teeth of the rigid fixed wheel (3) - in the diametrically opposite areas located near the major axis of symmetry of the waves generator, I-I), respectively of inner teeth of the flexible wheel in gaps between teeth rigid mobile wheel (4) - in the areas near the small axis of symmetry of the waves generator, II-II).

In order to ensure relative motion between the flexible toothed wheel and rigid fixed wheel, respectively between the flexible toothed wheel and rigid mobile wheel, it is necessary to correlate the number of teeth of the conjugated wheels, according to the following fitting condition:

$$z_3 - z_2 = k \cdot n_u ; \quad z_2^{/} - z_4 = k \cdot n_u \qquad (1)$$

where: n_u - is the number of waves deformation (n_u = 2); z - number of teeth of the wheel; k - constant (k = 1,2,3,…).

The transmission ratio of the DHGT is determined with R. Willis's relation [9]:

$$i_{14}^{(3)} = \frac{\omega_1}{\omega_4} = \frac{i_{34}^{(1)}}{i_{34}^{(1)} - 1} = \frac{z_2 \cdot z_4}{z_2 \cdot z_4 - z_2' \cdot z_3} \qquad (2)$$

where: ω_1 - is the angular velocity of the waves generator; ω_4 - the angular velocity of the rigid mobile wheel.

Lower values of the transmission ratio (i = 40…50) are obtained for constructive variant with the two teeth of the flexible wheel (external and internal) having the same number of teeth ($z_2 = z'_2$). In this case, the transmission ratio is calculated with the following relation:

$$i_{14}^{(3)} = \frac{z_4}{z_4 - z_3} \qquad (3)$$

3. Transmission ratio of the DHGT

The determination of transmission ratio of the DHGT can also be done using a different reasoning, which takes into account the dynamics of the flexible toothed wheel. It this case, we acknowledge the hypothesis that the flexible wheel keeps constant length of the dynamic reference fiber after elastic deformation.

For the kinematic analysis of DHGT, we represented the projections of the two drive zones, positioned at 90°, in a median cross section (A - A) of the DHGT, indicated by rolling (reference) fibers of the component wheels (Figure 2).

We observe that the flexible toothed wheel (2), when undeformed, the fixed rigid toothed wheel (3), and the mobile rigid wheel (4), have as a dynamics reference fiber some concentric circles. After mounting the waves generator, the dynamics reference fiber of the flexible wheel initially circle, has an elliptical shape.

When rotating the waves generator with an central angle φ_1, the dynamics reference fiber of the flexible wheel (2) will roll over without slipping on the circular dynamics reference fiber of the fixed rigid wheel (3), so that the point M will describe the trajectory M_0M. Simultaneously, the dynamics reference fiber of flexible wheel will be in contact with reference circle of the mobile rigid wheel (4), in areas that are at 90° to the major axis of symmetry of the ellipse (Figure 2).

We observe that the points M, respectively M', on the dynamics reference fiber of the flexible toothed wheel are describing the M_0M curve, respectively M'_0M', in fixed system of coordinate axis S(xOy), when the major axis of the waves generator (OY) rotates in clockwise direction by a φ_1 angle from the vertical direction.

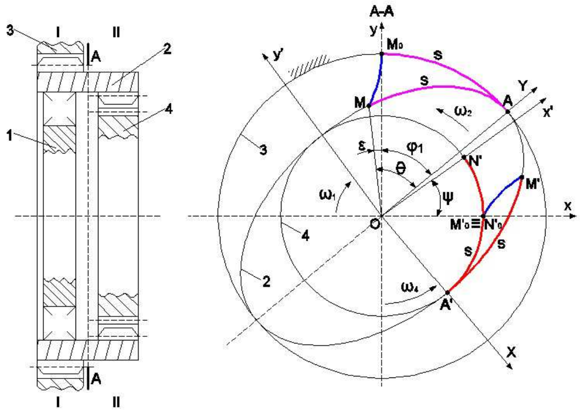

Figure 2. Kinematic model of the DHGT, variant $z_2 = z'_2$

Simultaneously with the trigonometric rotation of the flexible wheel by a ε angle, the mobile rigid wheel will rotate in the same direction by a ψ angle to the horizontal direction. Geometric arcs noted with AM_0, AM, A'M' and A'N', have the same length, noted by s, and arc length MM'' will represent a quarter of the entire length of the dynamics reference fiber of the flexible toothed wheel.

For transmission ratio of the DHGT, when $z_2 = z'_2$, we obtained the same expression as in relation (3):

$$i_{14}^{3} = \frac{\varphi_1}{\psi} = \frac{-AM_0/r_3}{M'_0N'/r_4} = \frac{-AM_0/r_3}{(A'N' - A'N'_0)/r_4} = \frac{-s/r_3}{\left(s - s\cdot\dfrac{r_4}{r_3}\right)\Big/r_4} = \frac{r_4}{r_4 - r_3} = \frac{z_4}{z_4 - z_3} \quad (4)$$

For the general case of the DHGT (Figure 3), when $z_2 \neq z'_2$, the transmission ratio realized in the first step of harmonic drive of the DHGT is:

$$i_{12}^{(3)} = \frac{\omega_1}{\omega_2} = \frac{\varphi_1}{\varepsilon} = \frac{\varphi_1}{\theta - \varphi_1} = \frac{-AM_0/r_3}{AM/r_2 - AM_0/r_3} = \frac{-s/r_3}{s/r_2 - s/r_3} = \frac{-r_2}{r_3 - r_2} = \frac{-z_2}{z_3 - z_2} \quad (5)$$

The ε angle of rotation of the flexible wheel, when the waves generator rotates with the φ_1 angle, is obtained from (5) in this form:

$$\varepsilon = \left(\frac{z_2 - z_3}{z_2}\right)\cdot\varphi_1 \quad (6)$$

Transmission ratio obtained in the second step of harmonic drive of the DHGT is:

$$i_{24}^{(1)} = \frac{\omega_2 - \omega_1}{\omega_4 - \omega_1} = \frac{\varepsilon - \varphi_1}{\psi - \varphi_1} = \frac{z_4}{z_2'} \;\Rightarrow\; \psi - \varphi_1 = \frac{z_2'}{z_4} \cdot \left(\varepsilon - \varphi_1\right) \tag{7}$$

By introducing the relation (6) in (7), we obtained dependence between the angles of rotation of the driving element (1) and the driven element (4):

$$\psi = \left(\frac{z_2 \cdot z_4 - z_2' \cdot z_3}{z_2 \cdot z_4} \right) \cdot \varphi_1 \tag{8}$$

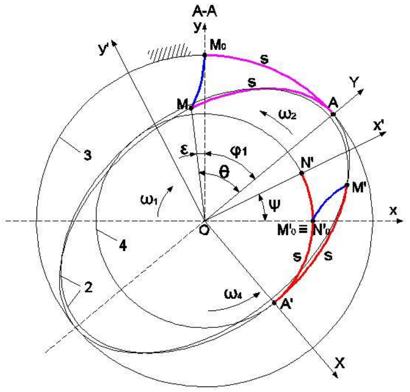

Figure 3. Kinematic model of the DHGT, variant $z_2 \neq z'_2$

The total transmission ratio of DHGT, for the variant $z_2 \neq z'_2$, will be:

$$i_{14}^{(3)} = \frac{\omega_1}{\omega_4} = \frac{\varphi_1}{\psi} = \frac{z_2 \cdot z_4}{z_2 \cdot z_4 - z_2' \cdot z_3} \tag{9}$$

We notice that, regardless of the reasoning method considered, geometric (9) or kinematic (2), we obtained the same expression for transmission ratio of DHGT.

4. Conclusions

In this paper we presented an original method of kinematic analysis of the DHGT, and we used it to calculate the transmission ratio for two constructive variants of these transmissions. The kinematics analysis emphasized the following conclusions:

- demonstrating the viability of the DHGT, by presenting the operating principle and constructive - functional peculiarities;
- determining of transmission ratio of the DHGT, by using the analytical method of the inversion of motion;
- the comparative analysis of transmission ratio for the two kinematic models of DHGT ($z_2 = z'_2$, respectively $z_2 \neq z'_2$).

References

[1] Anghel Ş., Ianici S., *Design of mechanic transmission,* Vol. I, Traian Vuia Politehnic Institute, Timişoara, 1993, pp. 265.

[2] Anghel Ş., Ianici S., *Proiectarea transmisiilor mecanice,* Vol. II, Editura Universitatea Tehnică Timişoara, 1993, pp. 233.

[3] Anghel Ş., Ianici S., Anghel C.V., *Îndrumar de proiectare a mecanismelor,* Universitatea Eftimie Murgu Reşiţa, 1994, pp. 120.

[4] Ianici S., Ianici D., *Testing of mechanical systems,* Editura Eftimie Murgu Reşiţa, 2010, pp. 121.

[5] Ianici S., Ianici D., *Constructive design and dynamic testing of the double harmonic gear transmission,* Analele Universităţii „Eftimie Murgu" Reşiţa, 2015, pp. 231-238.

[6] Ianici D., Ianici S., *Dynamic research of the flexible wheel of a double harmonic gear transmission,* Analele Universităţii „Eftimie Murgu" Reşiţa, 2015, pp. 223-230.

[7] Ianici S., Ianici D., *Elemente de inginerie mecanică,* UEM Reşiţa, 2015, pp. 251.

[8] Ianici S., Ianici D., *Design of mechanical systems,* Editura Eftimie Murgu Reşiţa, 2010, pp. 342.

[9] Ianici D., *Contributions to the constructive-functional improvement of the double gear harmonic transmission*, Ph.D. thesis, „Eftimie Murgu" University of Resita, 2012, pp. 202.

[10] Ianici S., *Contributions au calcul des deformations et tensions dans l´element flexible de la transmission harmonique hermetique*, Eighth IToMM Symposion SYROM 2001, Bucharest, pp. 175-180.

[11] Ianici S., *Contributions on determining the forces distributions in the toothed harmonic drive (THD),* The 8-th International Symposium MTM, Timişoara, 2000, pp. 165-168.

[12] Ianici S., Ianici D., a.o., *Double harmonic transmission used in industrial robotic drives*, Proceedings of International Conference on Robotics, No.1/2004, University of Timişoara, pp. 89-90.

[13] Nedelcu D., Ianici D., a.o., *The aerodynamic force calculus for a plate immersed in a uniform air stream using solidworks flow simulation module,* Proceeding of the 4th WSEAS, Paris, 2012, France, pp. 98-103.

Addresses:

- Lect. Dr. Eng. Draghiţa Ianici, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, d.ianici@uem.ro
- Prof. Dr. Eng. Sava Ianici, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, s.ianici@uem.ro